Crossover scaling of growing surfaces with threshold

1996 J. Phys. A: Math. Gen. 29 2651

(http://iopscience.iop.org/0305-4470/29/11/006)

J. Phys. A: Math. Gen. **29** (1996) 2651–2655. Printed in the UK

Crossover scaling of growing surfaces with threshold

Youngki Lee† and In-mook Kim‡§

Department of Physics, Korea University, Seoul, 136-701, Korea

Received 14 November 1995, in final form 1 February 1996

Abstract. We investigate a crossover behaviour of the conservative and non-conservative growing models in which the surface relaxation is restricted by the given value of threshold, H. The time dependence of the mean step height, which scales as $s(H,t) \sim Hg(t/H^2)$, is found to be responsible for the observed crossover behaviour of the surface width. A modified scaling ansatz is also proposed to describe the crossover behaviour in the growing models of this kind.

The kinetic roughening of a growing surface has attracted considerable interest during the past decade ([1] and references therein). A stochastically growing surface by the deposition of particles on an initially flat surface exhibits self-affine fractal structure and non-trivial scaling behaviour. Such scaling behaviour of the self-affine surface has been successfully described by the following dynamic scaling form for the surface width, W, developing in $(1+1)$-dimensional space [2]:

$$W(L,t) \equiv \left[\left\langle \frac{1}{L}\sum (h_i - \langle h\rangle)^2 \right\rangle\right]^{1/2} \sim L^\alpha f(t/L^z) \tag{1}$$

where h_i, L and t are the height of substrate site i, the lateral system size and the growth time, respectively. The symbol $\langle\cdots\rangle$ stands for the statistical average. The scaling function f has the asymptotic behaviour $f(x) \sim x^\beta$, for $x \ll 1$, and $f(x) \sim$ constant for $x \gg 1$. As a result, for short times ($t < t_s$), the surface width W depends only on time with the growth exponent β; $W(t) \sim t^\beta$, while for $t > t_s$, the saturated width depends only on the lateral system size with the roughness exponent α; $W(L) \sim L^\alpha$. The saturation time scales as $t_s \sim L^z$ with the dynamic exponent $z = \alpha/\beta$. These exponents characterize the kinetic roughening of the growing surface. One may address three different classes from the continuum equation proposed by Kadar, Parisi and Zhang (KPZ) [3];

$$\frac{\partial h(x,t)}{\partial t} = \nu\nabla^2 h(x,t) + \lambda(\nabla h)^2 + \eta(x,t) \tag{2}$$

where the surface height $h(x,t)$ is a function of t and substrate coordinates x, and $\eta(x,t)$ is the uncorrelated white noise. The case $\nu = \lambda = 0$ gives the random deposition ($\beta = \frac{1}{2}$; RD class). For $\nu \neq 0$ and $\lambda = 0$, it is the linear equation of the Edwards–Wilkinson model ($\beta = \frac{1}{4}$, $\alpha = \frac{1}{2}$ and $z = 2$; EW class), and for $\nu \neq 0$ and $\lambda \neq 0$, this nonlinear equation represents the KPZ class with $\beta = \frac{1}{3}$, $\alpha = \frac{1}{2}$ and $z = \frac{3}{2}$.

† E-mail address: youngki@kuccnx.korea.ac.kr
‡ E-mail address: imkim@kuccnx.korea.ac.kr
§ Author to whom all correspondence should be addressed.

0305-4470/96/112651+05$19.50

Some years ago Nagatani [4] introduced a threshold to the Edwards–Wilkinson [5] model to investigate the crossover behaviour from the random deposition (the RD class) to the deposition with surface diffusion (the EW class). In this restricted Edwards–Wilkinson (REW) model, the surface diffusion processes occur at the sites only where the step height between nearest-neighbour columns is greater than the given value of threshold H. Asymptotically, the surface width scales as $t^{1/2}$ in the limit of $H \to \infty$, but scales as $t^{1/4}$ in the limit of $H \to 0$. Therefore a crossover phenomenon is expected to be observed for a finite value of H, and a new characteristic time such as a crossover time t_c is also expected to appear in addition to the usual saturation time t_s. Nagatani (see [4]) reported that such a crossover time scales as $t_c \sim H^\phi$ with $1/\phi = 0.58$ for the REW model.

In this paper we reconsider the REW as a conservative growth, in which a crossover from the RD class to the EW class occurs. We also revisit the restricted solid-on-solid (RSOS) model [6] as a non-conservative growth, especially with higher restriction parameters rather than one, in which the crossover behaviour from the RD class to the KPZ class is expected to occur. In order to study the crossover behaviour of these restricted models, we apply the following dynamic rules for a randomly chosen site;

$$\begin{aligned} &h_i \to h_i + 1 && \text{if} \quad h_i - h_{i\pm1} < H \\ &h_i \to h_i && \text{otherwise for RSOS} \\ &h_{i\pm1} \to h_{i\pm1} + 1 && \text{otherwise for REW}. \end{aligned} \tag{3}$$

We then obtain the surface width by numerical simulations. All numerical results in our calculations are averaged over 1000 different configurations by varying the system sizes L and the values of threshold H. Time is counted by $t = N/L$ where N is the total number of attempts to deposit particles. A periodic boundary condition is also imposed in simulations.

We first consider the local state of a surface when a threshold H is introduced to the growing processes. The surface diffusion process in the REW model or desorption process in the RSOS model could not occur as long as the step height difference between the nearest-neighbour columns is smaller than the given value of H. Thus the random filling processes continue to develop until a statistically intrinsic length is comparable to that of the threshold. We can see this by investigating the time evolution of mean step height defined by

$$s(H,t) \equiv \langle |(h_i(t) - h_{i-1}(t)| \rangle \tag{4}$$

which is also the nearest-neighbour height–height correlation function. The dynamical scaling law (1) only holds if $s(H,t)$ is constant.

Figure 1 clearly shows that the mean step height, $s(t)$, increases as a power law, and saturates to an L independent value. The value of threshold H is thus a natural unit for measuring the mean step height. Associated with it is also the characteristic saturation time, t_c, which turns out to be the crossover time as will be seen below. This can be understood by mapping the dynamics of a step height to a one-dimensional random walk with two symmetric reflecting boundaries located at $\pm H$ site apart from the origin. Since the walker's first visit to the boundary means the occurrence of diffusion or desorption of a particle, one can identify the first passage time as the crossover time t_c and thus easily obtain $t_c \sim H^2$ from the well known random walk results. After a long run, the average distance of the walker, i.e. the mean step height, enters the steady state as $s(H,\infty) \sim H/2$. Evidently there is no L dependence of t_c. Hence we arrive at the scaling ansatz for the mean step height as follows:

$$s(H,t) \sim H^\gamma g(t/H^\phi) \tag{5}$$

where the scaling function g behaves asymptotically $g(x) \to x^{1/2}$ for $x \ll 1$ and $g(x) \to$ constant for $x \gg 1$. The insets of figure 1 confirm that all curves both of the

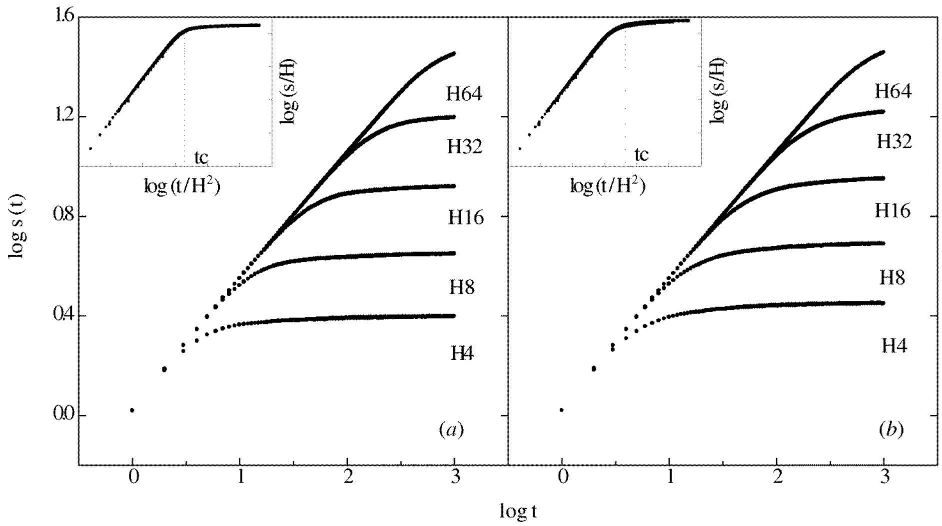

Figure 1. Plots of the mean step height against time with $H = 4, 8, 16, 32, 64$ and $L = 1000$, in log–log scale for the REW model (*a*) and the RSOS model (*b*). The insets show the data collapse according to the scaling equation (5), with $\gamma = 1.0$ and $\phi = 2.0$.

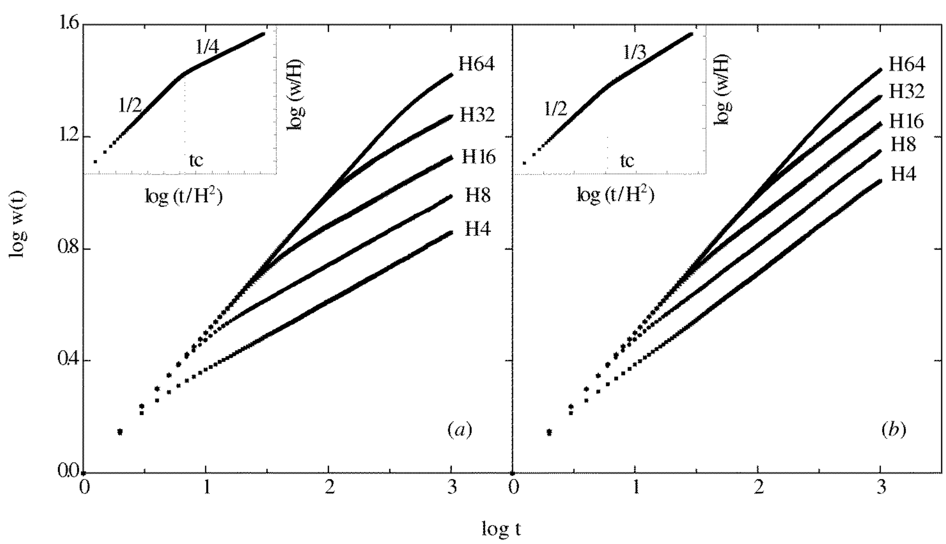

Figure 2. Plots of the surface width against time with $H = 4, 8, 16, 32, 64$ and $L = 1000$, in log–log scale for the REW model (*a*) and the RSOS model (*b*). The insets show the data collapse with W/H versus t/H^2.

REW (figure 1(*a*)) and the RSOS (figure 1(*b*)) are well collapsed to a single curve by the above scaling form with $\gamma = 1.0$ and $\phi = 2.0$.

Figure 2 shows the typical crossover behaviour of the surface width, $W(t)$, for the various values of threshold. As shown in the insets, all curves are excellently collapsed to a single curve with the slope, i.e. the growing exponent β, changing from $\frac{1}{2}$ to $\frac{1}{4}$ for the REW (figure 2(*a*)) and from $\frac{1}{2}$ to $\frac{1}{3}$ for the RSOS (figure 2(*b*)) at the crossover time $t_c \sim H^2$,

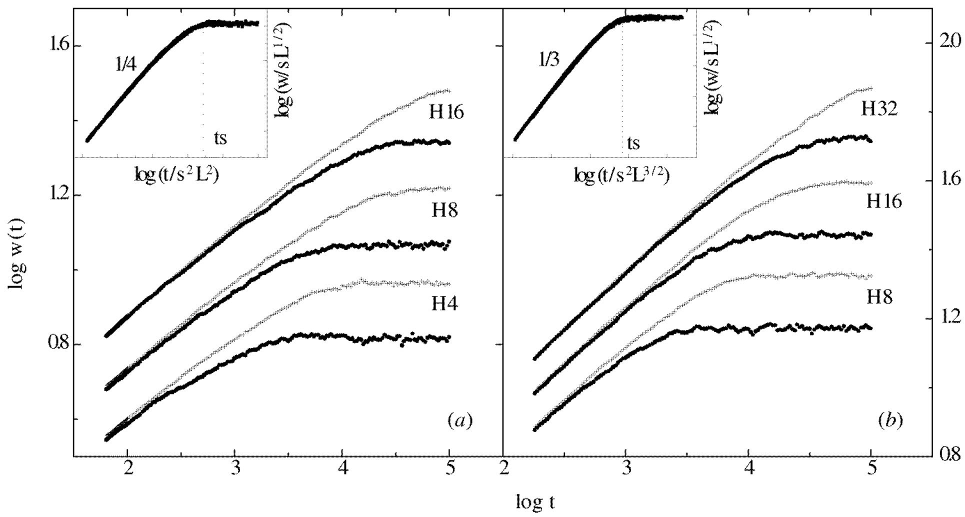

Figure 3. Log–log plots of the surface width with $H = 4, 8, 16$ for the REW model (a), and with $H = 8, 16, 32$ for the RSOS model (b). The system sizes are $L = 64$ (+ symbol) and $L = 128$ (full circle) for both cases. The insets show the data collapse according to the modified scaling equation (6), with (a) $\alpha = \frac{1}{2}$, $z = 2$ and (b) $\alpha = \frac{1}{2}$, $z = \frac{3}{2}$.

which is nothing but the characteristic time of the mean step height. Thus the power-law time dependence of $s(t, H)$ is responsible for the observed crossover behaviour of this kind of growing model. We also would like to point out that the value of $1/\phi = 0.58$ in [4] should be read with $\phi = 2.0$ as obtained in our scaling form.

We also investigate the long-time behaviour of the surface width for the time after t_c. Our simulations show that the saturation time scales as $t_s \sim H^2 L^z$ and the saturated surface width as $W_s \sim HL^\alpha$, for arbitrary values of L and H. We thus plot the rescaled surface width, W/W_s, against the rescaled time, t/t_s, in the log–log scale as shown in the insets of figure 3. The results show an excellent data collapse with exponents of the EW class (figure 3(a)) and those of the KPZ class (figure 3(b)). Its scaling form is given by the conventional scaling law as expected when $s(H, t)$ is constant. The scaling form unified by equations (1) and (5), which can describe the dependence of the surface width on the threshold, the system size and time, is proposed to be

$$W(H, L, t) \sim s(H, t) L^\alpha f(s^{-2}(H, t) t / L^z) . \qquad (6)$$

This implies that the surface width behaves as $W \sim t^{1/2}$ for $t < t_c$, $W \sim H^{1-2\beta} t^\beta$ for $t_c < t < t_s$, and $W \sim HL^\alpha$ for $t_s < t$, which is consistent with all simulation results. This type of scaling form is not unfamiliar. Schroeder *et al* [7] recently introduced a modified scaling law to account for an anomalous scaling behaviour arising in the Wolf–Villian [8] model. They observed the time dependence of the mean step height and introduced a modified scaling relation for the height–height correlation function. The direct integration of their scaling relation easily gives the above equation (6). This modified scaling relation successfully explains the complex scaling behaviour of $W(L, H, t)$ in all time regimes for the various system sizes and thresholds. Recently, Nattermann and Tang [9] studied the continuum equation (2) in the weak-coupling regime and discussed the crossover scaling of the surface width. Their analytic description could be followed by providing that the square of the restriction parameter in our model is reciprocal to the diffusion constant.

In summary, we have studied the crossover behaviour of a conservative and non-conservative growing surface in the presence of a threshold. The time dependence of the mean step height $s(H, t)$ is found to be responsible for the observed crossover behaviour. We have examined the REW and RSOS models as an example of a conservative and non-conservative growing surface, respectively. In both models, the mean step height grows as $t^{1/2}$ and then saturates to $H/2$, and scales as $s(H, t) \sim Hg(t/H^2)$. There also exist two characteristic times $t_c \sim H^2$ and $t_s \sim L^z$. The crossover time t_c turns out to be the saturation time of the mean step height. We also propose a modified scaling relation to describe a complex scaling behaviour of $W(L, H, t)$ in all time regimes for the various system sizes and thresholds. In principle, equation (6) can be applied to a model if an existing crossover is induced by the time dependence of the mean step height. Especially when the crossover time is close to the saturation time such as $t_s \sim t_c$, one cannot then obtain a correct behaviour of the surface width by using a conventional scaling law, since it begins to saturate before the crossover occurs. In such a case, our modified scaling ansatz makes it possible to determine the correct universality class of the system. The applicability to other growth models, in which the crossover occurs by non-trivial dynamics rather than random dynamics, is now under investigation and shall be reported elsewhere.

Acknowledgments

We would like to thank Professor J M Kim, Hallym University, for critical discussions and reading the manuscript. This work is supported in part by the Ministry of Education (no BSRI-95-2409), KOSEF (no 951-0206-003-2), and also by the Korea Research Centre of Theoretical Physics and Chemistry.

References

[1] Family F and Vicsek T 1991 *Dynamics of Fractal Surfaces* (Singapore: World Scientific)
Barabási A-L and Stanley H E 1995 *Fractal Concepts in Surface Growth* (Cambridge: Cambridge University Press)
[2] Family F and Vicsek T 1985 *J. Phys. A: Math. Gen.* **18** L75–81
[3] Kardar M, Parisi G and Zhang Y-C 1986 *Phys. Rev. Lett.* **56** 889–92
Medina E, Hwa T, Kadar M and Zhang Y-C 1989 *Phys. Rev.* A **39** 3053–75
[4] Nagatani T 1990 *J. Phys. A: Math. Gen.* **23** L1139–43
[5] Edwards S F and Wilkinson D R 1982 *Proc. R. Soc.* A **381** 17–31
[6] Kim J M and Kosterlitz J M 1989 *Phys. Rev. Lett.* **64** 543–6
[7] Schroeder M, Siegert M, Wolf D E, Shore J D and Plischeke M 1993 *Europhys. Lett.* **24** 563–8
[8] Wolf D E and Villain J 1990 *Europhys. Lett.* **13** 389–94
[9] Nattermann T and Tang L H 1992 *Phys. Rev.* A **45** 7156–61